# **A**nswers and Explanations

1	2	2	1	3	3	4	5	5	4	6	3	7	1	8	4	9	2	10	5
11	4	12	2	13	1	14	4	15	3	16	1	17	4	18	1	19	5	20	2
21	2	22	1	23	3	24	5	25	2	26	3	27	5	28	1	29	4	30	2
31	2	32	1	33	2	34	4	35	1	36	4	37	3	38	5	39	4	40	4
41	1	42	3	43	3	44	2	45	5	46	5	47	2	48	3	49	5	50	3
51	1	52	5	53	3	54	1	55	2	56	5	57	5	58	5	59	1	60	3
61	4	62	2	63	1	64	5	65	2	66	3	67	4	68	3	69	1	70	1
71	4	72	4	73	5	74	2	75	1	76	3	77	3	78	2	79	3	80	3
81	2	82	4	83	1	84	5	85	4	86	2	87	3	88	1	89	5	90	4

### For questions 1 to 5:

From Dream Team 7:  $\rm P_2$  and  $\rm P_9$  belong to LA Lakers. So  $\rm P_{12}$  cannot be from LA Lakers.  $\rm P_{12}$  is not from Utah Jazz. So  $\rm P_{12}$  must be from Chicago Bulls.

**From Dream Team 6:** P<sub>7</sub> and P<sub>12</sub> belong to Chicago Bulls. Now in dream team 7, P<sub>15</sub> cannot belong Chicago Bulls, because then in dream team 6, there would be three players who belong to Chicago Bulls, which is not possible. So, P<sub>15</sub> belongs to Utah Jazz (from dream team 7) and P<sub>14</sub> belongs to Chicago Bulls. Now P<sub>6</sub> and P<sub>3</sub> belong to LA Lakers because in dream team 6, P<sub>7</sub> and P<sub>12</sub> belong to Chicago Bulls and P<sub>15</sub> belongs to Utah Jazz.

From dream team 1:  $P_1$  belongs to Utah Jazz because  $P_3$  and  $P_9$  belong to LA Lakers and  $P_7$  and  $P_{12}$  belong to Chicago Bulls.

From dream team 4:  $P_{10}$  belongs to Chicago Bulls because  $P_2$  and  $P_6$  belong to LA Lakers and  $P_7$  and  $P_1$  belong to Chicago Bulls and Utah Jazz respectively.

From dream team 2: If  $\rm P_{13}$  belongs to Chicago Bulls and  $\rm P_{11}$  belongs to Utah Jazz, or vice-versa. Assume that  $\rm P_{11}$  is from Utah Jazz.

But then in dream team 5 we have two players,  $P_1$  and  $P_{11}$  belonging to Utah Jazz, which is not possible; because in every dream team there is only one player from Utah Jazz.

Therefore  $P_{11}$  belongs to Chicago Bulls and  $P_{13}$  belongs to Utah Jazz. (From dream team 2)

From dream team 3: P<sub>5</sub> belongs to Utah Jazz.

From dream team 8:  $\rm P_4$  and  $\rm P_8$  belong to LA Lakers as  $\rm P_{10}$  and  $\rm P_{11}$  belong to Chicago Bulls and  $\rm P_{13}$  belong to Utah Jazz.

From dream team 5:  $P_{16}$  belongs to LA Lakers because  $P_{10}$  and  $P_{11}$  belong to Chicago Bulls and  $P_1$  belongs to Utah Jazz.

Now, the following conclusion can be drawn **Chicago Bulls (5 players):** P<sub>7</sub>, P<sub>10</sub>, P<sub>11</sub>, P<sub>12</sub> and P<sub>14</sub>. **LA Lakers (7 players):** P<sub>2</sub>, P<sub>3</sub>, P<sub>4</sub>, P<sub>6</sub>, P<sub>8</sub>, P<sub>9</sub> and P<sub>16</sub>. **Utah Jazz (4 players):** P<sub>1</sub>, P<sub>5</sub>, P<sub>13</sub> and P<sub>15</sub>

- 1.2 Four players namely  $P_1$ ,  $P_5$ ,  $P_{13}$  and  $P_{15}$  belong to Utah Jazz.
- 2.1 Seven players namely P<sub>2</sub>, P<sub>3</sub>, P<sub>4</sub>, P<sub>6</sub>, P<sub>8</sub>, P<sub>9</sub> and P<sub>16</sub> belong to LA Lakers.

- 3.3  $P_4$ ,  $P_8$ ,  $P_{16}$ ,  $P_{13}$  and  $P_{15}$  are the players in the team formed by Mr. XYZ. In this team  $P_4$ ,  $P_8$  and  $P_{16}$  are from LA Lakers and  $P_{13}$  and  $P_{15}$  are from Utah Jazz. Total match fee = 775 x 3 + 725 x 2 = \$ 3775.
- 4.5 The team that had P<sub>13</sub>, P<sub>16</sub>, P<sub>14</sub>, P<sub>11</sub> and P<sub>4</sub> has two players each from LA Lakers (P<sub>16</sub> and P<sub>4</sub>) and Chicago Bulls (P<sub>14</sub> and P<sub>11</sub>) and P<sub>13</sub> (from Utah Jazz). This team earns the maximum aggregate match fee for each game played by them.
- 5. 4 In the team, P<sub>13</sub>, P<sub>4</sub>, P<sub>11</sub>, P<sub>14</sub>, P<sub>16</sub> there are two players each from Chicago Bulls and LA Lakers and one player from Utah Jazz and hence is not a possible team that had at least two players from Utah Jazz and one player each from Chicago Bulls and LA Lakers.

## For questions 6 to 10:

- 6.3 One '0' starts and next '0' stops the machine. For the machine to be in stop position ultimately, number of times '0' is present in the input should be even. Only option (3) doesn't have such provision.
- 7. 1 A starting '0' is given to start the machine. After that, if the machine stops, it must start immediately before getting '1' to produce the product. Hence, besides the first and last '0', all intermediate 0's should exist in pairs. Input in option (1) is the only exception.
- 8.4 Breaking the input strings into two-two bits,
  - I. 00 10 10 10 10 10 10 10 10 10 11 11 01 10 01 10 00 01 01 01 01 01

Thus, it can be seen that

	Units of A produced	Units of B produced
	5	9
=	3	3
	7	2

9.2 In the previous question, 2 different kinds of products along with start instruction and stop instruction are 4 different processes that the machine can execute.

For these 4 processes, 2 bits were required. Similarly with 3 bits, combinations **000**, **001**, **010**, **011**, **100**, **101**, **110**, **111** can make the machine execute 8 different processes. Therefore, for 1802 processes including start and stop, we require at least 11 bits, because  $2^{11} < 1802$ .

10.5 Take an example.

Input = **01101**.

Checking option (1), parity bit is initially '0'. Then, it gets reversed 3 times (because of three occurrences of 1) to become '1' ultimately. Now, since at most 1 bitreversal can happen, it may happen that one of the '0' becomes '1' (total number of 1 = 4) or one of the '1' becomes '0' (total number of 1 = 2). In either of the case, recomputation of the parity would not yield the same result. Thus, error would get caught. Same is true for each of the other options (2), (3) and (4).

### For questions 11 to 15:

- 11.4 There can be two cases when Sanya paid Rs. 35 more.
  - Case I: Sanya ordered for 2 plates of Dosa, 3 plates of Pizza, 2 plates of Berger and 2 plates of Idli in the dinner party hosted by her and Ravanya ordered for 1 plate of Dosa, 2 plates of Pizza, 3 plates of Berger and 3 plates of Idli in the dinner party hosted by her.

So Ravanya paid Rs. 220.

**Case II:** Sanya ordered for 1 plate of Dosa, 3 plates of Pizza, 2 plates of Berger and 3 plates of ldli in the dinner party hosted by her and Ravanya ordered for 3 plates of Dosa, 1 plate of Pizza, 3 plates of Berger and 2 plates of Idli in the dinner party hosted by her.

So Ravanya paid Rs. 210.

12. 2 The maximum amount will be reached when 3 plates of Dosa, 3 plates of Pizza, 2 plates of Berger and 1 plate of Idli will be ordered. It could have been from Piyashi, Ravanya, Sanya and Varsha because only they have ordered for 9 items. But on Monday only Piyashi can host the dinner party among them, which is obvious from the following table collating all possible sequences of hosting the parties.

	Mon.	Tue.	Wed.	Thu.	Fri.	Sat.	Sun.
Case 1	Р	R	S	V	J	q	Т
Case 2	Р	R	S	V	Т	U	Q
Case 3	Т	Р	R	S	V	U	Q
Case 4	U	Q	Р	R	s	V	Т
Case 5	Т	U	Q	Р	R	S	V
Case 6	U	Q	Т	Р	R	S	V

- 13. 1 In the dinner party hosted by Qualin, she ordered for 2 plates of Dosa, 2 plates of Pizza, 2 plates of Berger and 2 plates of Idli. She paid Rs. 210. Some other friend paid minimum amount when 2 plates of Dosa, 1 plate of Pizza, 2 plates of Berger and 3 plates of Idli was ordered by her. Total amount in this case was Rs.180.
  So difference = (210 180) = Rs. 30.
- 14. 4 Referring to the table given in the explanation of question 12, there are six possible sequence. We can see that on Wednesday and Friday, 5 friends can possibly host the party. Among the given options, (4) Friday is the right answer.

15.3 If only Urvashi, Qualin and Piyashi ordered for 2 Bergers each in the parties hosted by them, then others had ordered for 3 Bergers each.

If only Qualin, Tanya, Piyashi, Ravanya and Varsha ordered for 2 Pizzas each in the parties hosted by them, then others had ordered for 3 Pizzas each.

If only Piyashi and Ravanya did not order for 2 plates of Dosa each in the parties hosted by them, then they ordered for either 3 plates of dosas or 1 plate of dosa. From information (IV), Piyashi, Ravanya, Sanya and Varsha must have ordered for 9 plates each and others have order for 8 plates each. After considering all the constraints, the following table can be collated:

Name	Dosa	Pizza	Berger	Idli
Р	3	2	2	2
Q	2	2	2	2
R	1	2	3	3
S	2	3	3	1
Т	2	2	3	1
U	2	3	2	1
V	2	2	3	2
Total	14	16	18	12

Ravanya must have ordered for 3 plates of Idli.

For questions 16 to 20:

	Parliament	State 1	State 2	State 3
Weight	6	3	2	1
Maximum possible member belonging to a party	297	110	198	66
Minimum possible member belonging to a party	243	90	162	54
Total	540	200	360	120

16. 1 Maximum Possible votes that one candidate can get =  $297 \times 6 + 110 \times 3 + 198 \times 2 + 66 \times 1 = 2574$ Minimum Possible votes that one candidate can get =  $243 \times 6 + 90 \times 3 + 162 \times 2 + 54 \times 1 = 2106$ So, the votes received by any candidate lies between 2106 and 2574. (both inclusive)

So the votes received by Ms. Pratt must be within this range. Clearly option (1) is not in this range.

17. 4 Since Mr. Shake wins the election we will take the composition of members in the elected councils such that party C enjoys maximum majority in each of the elected councils.

	Parliament	State 1	State 2	State 3	
Weight	6	3	2	1	
Party C	297	110	198	66	
Party B	243	90	162	54	
Total	540	200	360	120	

all the council members of party C in states 2 and 3 abstain from voting, then the difference between total votes (after due weight) of party C and party B is (54  $\times$  6 + 20  $\times$  3) - (162  $\times$  2 + 54  $\times$  1) = 384 - 378 = 6. Now if another three council members of party C in state 1 abstain from voting then the total votes (after due weight) of party B is 9 - 6 = 3 more than the total votes (after due weight) of party C and hence Mr. Shake wins the election.

Therefore minimum possible number of council members who should definitely abstain from voting such that Mr. Shake still wins the election = 198 + 66 + 3 =267.

18. 1 **Case I** : Difference between the council members of party B and party C is minimum possible.

	Parliament	State 1	State 2	State 3
Weight	6	3	2	1
Party C	273	99	179	59
Party B	267	101	181	61
Total	540	200	360	120

Therefore, Y = 2 + 2 + 2 - 6 = 0

**Case II** : Difference between the council members of party B and party C is maximum possible.

	Parliam ent	State 1	State 2	State 3
Weight	6	3	2	1
Party C	271	90	162	54
Party B	269	110	198	66
Total	540	200	360	120

Therefore, X = 12 + 36 + 20 - 2 = 66 $\therefore (X - Y) = 66 - 4 = 62$ 

19. 5 Mr. Karl got all the votes from the Assemblies. That means he already got (200 x 3 + 360 x 2 + 120 x 1) = 1440 votes.

The maximum number of MPs, who voted for the candidates supported by their respective parties, such that Mr. Karl still manages to win the election, will be in the following case:

	Party B	Party C	Mr. Karl
Number of MPs voted for	260	259	21
Value of the vote after due weight	1560	1554	126

In this case Karl gets (1440 + 126) = 1566 votes, and wins the election.

The maximum possible number of MPs, who voted for the candidates supported by their respective parties is (260 + 259) = 519

20. 2 For maximum possible difference in votes (after due weight) the possible combination of states in which one party enjoys majority is state 1 and state 3 and the other party enjoys majority in the parliament and state 2.

Maximum possible difference in votes (after due weight) =  $(54 \times 6 + 36 \times 2) - (2 \times 3 + 2 \times 1) = 396 - 8$  = 388.

# For questions 21 to 25:

Let a,b,c,d be the number of days on which those four employees were present. (a > b > c > d)

 $\Rightarrow$  a + b = 51, c + d = 40 and a + c = 49

$$a + b = 51 \implies a \ge 26$$
 and  $b \le 25$ 

 $c + d = 40 \implies c \ge 21 \text{ and } d \le 19$ 

a + c = 49  $\Rightarrow$  a  $\geq$  25 and c  $\leq$  24

Thus, 'c' could be 21 or 22 or 23 or 24. Corresponding values of a,b,d would be

Cases	Ι	I	Ξ	IV
а	28	27	26	25
b	23	24	25	26
С	21	22	23	24
d	19	18	17	16

But in the rightmost case IV, a<b. Thus, it may be ignored.

Cases	Ι	=	=
а	28	27	26
b	23	24	25
c	21	22	23
d	19	18	17

- 21. 2 Column figure numbered 5 = b + d = (a + b) + (c + d) (a + c)= 51 + 40 - 49 = 42
- 22. 1 From the table above, the only way two figures can sum up to 45 is (a + d) in case II. Thus, the column figure 3 should read 46.
- 23. 3 Eksa was present on 'c' days. Maximum value of c = 23.
- 24. 5 Column figure numbered 3 is (a + d). Thus, the other two columns must be (b + c) and (b + d). Also, since (a + d) is column figure numbered 3, a + d > b + c This is happening only for case I.
  (a + d) = 47
- 25. 2 Let x, y, z and w be the number of days on which exactly one, exactly two, exactly three and exactly four of these mentioned employees were present. Therefore, x + 2y + 3z + 4w = 91 ...(i) Also, x + y + z + w = 30 ...(ii)

**Case I:** y = 0  $\Rightarrow 2z + 3w = 91 - 30 = 61$ . Possible values of x, z and w are tabulated below:

x	z	w
9	2	19
8	5	17
7	8	15
6	11	13
5	14	11
4	17	9
3	20	7
2	23	5
1	26	3
0	29	1

**Case II:** z = 0Possible values of x, y and w are tabulated below:

x	у	w
7	4	19
5	7	18
3	10	17
1	13	16

**Case III:** x = 0Possible values of y, z and w are tabulated below:

у	z	w
0	29	1
1	27	2
2	25	3
3	23	4
4	21	5
5	19	6
6	17	7
7	15	8
8	13	9
9	11	10
10	9	11
11	7	12
12	5	13
13	3	14
14	1	15

Therefore, the number of days on which exactly 3 employees were present can never be equal to 4. Hence option (2) is the correct choice.

# For questions 26 to 30:

Let us define the events 'Bridge the Gap', 'Dragon Lake', 'Fortress', 'Giant Maze' and 'Muddy Waters' as events 1, 2, 3, 4 and 5 respectively.

On Monday there are 2 rollover participants in event 1. They will participate in the same event on Tuesday. So new participants on Tuesday in the event 1 = 28 - 2 = 26

On Tuesday in event 2, 22 participants take part, but out of them 3 are rollover participants from Monday. So on Tuesday the number of fresh participants reaching event 2 = 22 - 3 = 19 So rollover participants in event 1 on Tuesday = 28 - 19 = 9 Similarly there are 6 rollover participants in event 3 from Monday.

So number of fresh participants reaching event 3 on Tuesday = 24 - 6 = 18So rollover participants in event 2 on Tuesday = 22 - 18 = 4

The following table shows the number of rollover participants on each day.

	Bridge the Gap	Dragon Lake	Fortress	Giant Maze
Mon	25–23 = 2	23–20 = 3	20–14 = 6	14–11 =3
Tue	28–(22–3) = 9	22–(24–6) = 4	24–(26–3) = 1	26–20 = 6
Wed	20–(18–4) = 6	18–(13–1) = 6	13–(13–6) = 6	13–13 = 0
Thu	25–(27–6) = 4	27–(20–4) = 11	20–(16–0) = 4	16–12 = 4
Fri	22–(18–11) = 15	18–(22–4) = 0	22–(24–4) = 2	24–22 = 2

26. 3 Maximum rollover participants (15) are in event 1 (Bridge the Gap) on Friday.

27.5 Fresh participants = total participants - rollover participants So maximum possible number of fresh participants in

different events on Tuesday are

- i) Bridge the gap = 28 2 = 26
- ii) Dragon Lake = 22 3 = 19iii) Fortress = 24 - 6 = 18
- iv) Giant Maze = 26 3 = 23.

However this is not possible as number of fresh participants in Fortress is only 18. So in Giant Maze number of fresh participants cannot be more than 18. So maximum ratio = 18:26 = 9:13

28.1 Rollover participants in different events on Tuesday and Wednesday are tabulated below:

	Bridge the Gap	Dragon Lake	Fortress	Giant Maze	
Tuesday	9	4	1	6	
Wednesday	2	5	0	0	

So if these participants again participate on Thursday, then rollover participants on Thursday are

	Bridge the Gap	Dragon Lake	Fortress	Giant Maze
Thursday	25 – [27–(4+5)]	27–(20–1)	20–(16–6)	16–12
	= 7	= 8	= 10	=4

So total rollover participants on Thursday = 7 + 8 + 10+ 4 = 29

- 29. 4 Maximum sum of rollover participants is on Thursday = 4 + 11 + 4 + 4 = 23
- 30. 2 Maximum variation = (Maximum number of rollover participants among the given five days Minimum number of rollover participants among the given five days) in a particular event
   So, Maximum variation = 15 2 = 13 which was in event 1.
- 31. 2 Statement D discusses the artist, C goes on to describe the critic, DC form a mandatory pair essentially through the association of comparison. This is followed immediately with a description of criticism in B. E goes on to extend the idea, A marks a closing comment.
- 32. 1 E is the opening statement. It goes on to specifics in C, B describes the work of Camus, D defends Camus' stance and A concludes the debate. Note that DA is a mandatory pair- linked by 'existentialists'.
- 33. 2 C is quite obviously the opener. D goes on to define the term. B & E take the issue further, A forms the conclusion. Note that DBE is a mandatory triad.
- 34. 4 D is the obvious opener, E describes it further, A makes a statement which is closely linked to statement B, C marks the grand closing statement. DE and AB are mandatory pairs.
- 35. 1 B makes the opening statement as it is the most general statement of the lot talking about 'price of education'. D goes on to describe the current status of education. E goes on to describe the scenario, C goes on to reiterate the same and A marks the closure by making a comment on the same. Note that BDE talk about what we have done and CA talk about wonder itself.
- 36. 4 "which he thought pointed to a supreme designer,...that idea was attacked by David Hume", these lines point out Hume's stance vis-à-vis Voltaire's, making option 4 the correct answer.
- 37. 3 Paragraph 2 highlights the strategy adopted by Hume in his writings, Refer the following lines "...recognized that only gentle and reassuring persuasion would work.", option (3) brings out this facet of his writing style very clearly making it the correct answer.
- 38.5 Hume is very clearly seen as a great strategist; all along he makes a fine show of piety whereas very subtly he is questioning the very rationale of religion in his writings. This makes option 5 the correct answer.

- 39. 4 The main reason for Hume's rejection was their-"intolerant zeal" and dogmas, making option (4) correct.
- 40. 4 The last paragraph deals mainly with the advent of atheism in the world and explores the possible reasons that have led to this scenario. Option (4) is the correct answer as it deals with some of the deeper religious experiences. It is a reason for religion to persist and not 'spread of unbelief'.
- 41. 1 Option (1) is correct. Taking a cue from 'the Socratic method', option (1) discusses the other method that is being taken up as a consequence of the discussion. Option (2) which is close seems like a repetition as collective unconscious has been explained earlier.
- 42. 3 Option (3) continues the tone and the theme of the paragraph in the best possible manner. 'More money means more votes' is anti-democratic which leads to (3) 'an unfair distribution'. (2) is already implied in the paragraph. It could follow (3). The paragraph talks about 'an individual'. (4) talks about a 'group' and also has some repetition of idea. (1) and (5) are related but disjointed in terms of idea.
- 43. 3 Option (3) follows immediately after the passage and begins by citing an example and then the other examples follow. Only (3) introduces the example the others directly jump to the examples and the past tense.
- 44. 2 The paragraph has spoken about what happens on a surface level. So, logically, digging below the surface would follow. Also, the paragraph ends with 'interpretation' and 'watching Kubrick's movie'. Option (2) extends this with what happens next 'speculation' following 'interpretation'. (1) abruptly jumps to the first person. (3), (4) and (5) talk about 'Kubrick' himself but do not extend the idea effectively at the end of the paragraph.
- 45. 5 Option (5) continues in the same breath as the passage. It moves the discussion forward, talking about the newspaper journalists - hence continuing or extending the idea.
- 46. 5 The lines from the passage " tackling work has fallen overwhelmingly to the realist novelist", makes option (5) correct.
- 47. 2 The Rise of Silas Lapham, is a saving grace with its display of moral rectitude. The 'rise' is seen in the passage as a 'moral reaffirmation'.
- 48. 3 The line "depiction of the moral relativity of commercial life and the supremacy of the individual's self-interest", makes option (3) correct.
- 49. 5 "Lewis unveils the stultifying conformity of mowed lawns and motorcars and conservative political views that, to his dismay, had gripped middle America between the world wars.' This statement clearly supports option (5) and supports his reason for creating an Everyman.
- 50. 3 The statement epitomizes the state of mind of the characters which in turn depicts the dissatisfaction present in the people of those times. The lines from the passage, "the characters in The Man in the Gray Flannel Suit and Revolutionary Road are woefully disaffected" makes Option (3) correct.

- 51. 1 A credulous person 'believes' easily he is gullible. Similarly a peevish person gets irritated easily. Avaricious means greedy, lecherous means lusty, Soporific is sleepy, Malevolent is harmful - not necessarily causing death.
- 52. 5 Minatory is threatening / menacing / evil which is more or less opposite of irreprehensible - morally good. Nugatory / Jejune is uninteresting; Callous means torpid, a Stentorian is consonant / resonant, stridulous means craggy. Prelection - discourse is opposite to harken listen.
- 53. 3 Three errors. Words like 'in his opinion', 'he thought' and 'back' are redundant and can be dropped from the sentence.
- 54. 1 One error. His application
- 55. 2 Two errors. Many linguists are not aware that one out of ten English speaking adults in the city lacks critical communication skills. 'Each' is redundant.
- 56. 5 Option (5) is the correct answer. The lines "... most importantly, they begin to remember which of their responses were effective in which contexts," supports the answer.
- 57. 5 In the passage the author cites the following, "They must learn within the already determined environment of the textbook to focus student attention on the key issues, which in linked sequence provide the essence of a stage of the mastery of a discipline", making option (5) correct.
- 58. 5 The lines, "Virgil's greatness as a guide and teacher for Dante rested in his understanding that his student must experience, either directly or vicariously, all the possibilities of the human soul before discussion would be of value", bring out the reason for Virgil's greatness, making option (5) correct.
- 59. 1 Option (1) is correct, the lines "Responding constantly to questions emerging from students' experience, teachers will re-assume the Socratic mantle and reverse the progressive de-skilling the profession" provide the right answer; nowhere does the author state that 'preparing' for travel will advance the reversal.
- 60. 3 Option (1) is incorrect as it covers the passage in part, option (2) gives predominance to 'old wine' which is incorrect, option (3) is most appropriate as it is a new approach dealing with new aspects even though it takes the help of some ancient examples. Option (4) is not specific as the passage is not just about teaching but specifically about the new methodology. Option (5) is too general.

61. 4 
$$\left|2-\frac{3}{x}\right| \le \frac{1}{3} \Rightarrow -\frac{1}{3} \le 2-\frac{3}{x} \le \frac{1}{3}$$
  
 $\Rightarrow \frac{9}{7} \le x \le \frac{9}{5}$   
 $\left|3-\frac{x}{y}\right| \le \frac{1}{6} \Rightarrow -\frac{1}{6} \le 3-\frac{x}{y} \le \frac{1}{6} \Rightarrow \frac{17}{6} \le \frac{x}{y} \le \frac{19}{6}$   
 $\Rightarrow \frac{y}{x} \ge \frac{6}{19} \text{ and } \frac{y}{x} \le \frac{6}{17}$ 

$$\Rightarrow \frac{6}{19} \le \frac{y}{x} \le \frac{6}{17}$$
Maximum value of  $\frac{x}{2 - \frac{y}{x}} = \frac{\frac{9}{5}}{2 - \frac{6}{17}} = \frac{153}{140}$ 
Minimum value of  $\frac{x}{2 - \frac{y}{x}} = \frac{\frac{9}{7}}{2 - \frac{6}{19}} = \frac{171}{224}$ 
 $\frac{171}{224} \le \frac{x^2}{2x - y} \le \frac{153}{140}$ 
Only  $\frac{\sqrt{3}}{2}$  lies within the range specified above

Let the speeds of Ayesha and Bhumika be 'a' and 'b' units respectively.

Since Bhumika started running at 0600 hrs and they met at 0700 hrs, the ratio of the distance AP : PB = 2a : b

It is also given that they met at the same point P, while coming back.

Thus, 2AP : 2PB = b : a

62. 2

$$\therefore \frac{2a}{b} = \frac{b}{a} \Rightarrow \left(\frac{a}{b}\right)^2 = \frac{1}{2} \text{ or } \frac{a}{b} = \frac{1}{\sqrt{2}}$$

63. 1 a: b =  $7\sqrt{3}$  : 14 =  $\sqrt{3}$  : 2

Thus, AP : PB = 2 :  $\sqrt{3}$ 

Let AP = 2k and PB =  $\sqrt{3} \times k$ 

By 0600 hrs, Ayesha must have covered  $7\sqrt{3}$  kms. From 0600 hrs, distance that Ayesha and Bhumika covered was in the ratio  $\sqrt{3}:2$ 

$$\Rightarrow \frac{(2k - 7\sqrt{3})}{(k \times \sqrt{3})} = \frac{\sqrt{3}}{2}$$
$$\Rightarrow k = 14\sqrt{3}$$

 $\Rightarrow$  AB = (2 +  $\sqrt{3}$  ) x k = (42 + 28  $\sqrt{3}$  ) km which is approximately equal to 90.5 km



Let the radii of the larger and smaller circle be  $\ensuremath{r_1}$  and  $\ensuremath{r_2}$  respectively.

Area of the annular ring =  $\pi (r_1^2 - r_2^2) = \pi (r_1 + r_2) (r_1 - r_2)$ 

Area of quadrilateral EBCH

= Area of  $(\Delta EOB + \Delta BOC + \Delta COH + \Delta HOE)$ 

$$= \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)^2$$
  

$$\therefore \text{ The ratio} = \frac{2\pi(\mathbf{r}_1)}{2\pi(\mathbf{r}_1)}$$

The ratio 
$$=\frac{2\pi(r_1 - r_2)}{(r_1 + r_2)} = \frac{3\pi}{2}$$
 (Given)

r) 2-

 $\Rightarrow$  r<sub>2</sub> : r<sub>1</sub> = 1:7

65. 2 Assume the numbers to be 13 x N and 17 x M, where N and M are two digit prime numbers. There are two possible ways in which 7 can be the unit's digit of the product i.e. 1 x 7 or 3 x 9.

**Case I:** If units digit of N is 3, then units digit of M will be 9.

Then, N = 13, 23, 43, 53, 73 or 83 and M = 19, 29, 59, 79, 89

So, number of distinct products =  $6 \times 5 = 30$ 

 $\mbox{Case II: If units digit of N is 1, then units digit of M will be 7 }$ 

Then N = 11, 31, 41, 61 or 71 and M = 17, 37, 47, 67 or 97  $\,$ 

So, number of distinct products =  $5 \times 5 = 25$ 

Case III: If unit digits of N is 7, then units digit of M will be 1.

Then N = 17, 37, 47, 67 or 97 and M = 11, 31, 41, 61 or 71  $\,$ 

So number of distinct products =  $5 \times 5 = 25$ 

 $\label{eq:case_loss} \begin{array}{l} \textbf{Case IV:} \ \text{If unit digit of N is 9, then unit digit of M will be } \\ \textbf{3.} \end{array}$ 

Then N = 19, 29, 59, 79 or 89 and M = 13, 23, 43, 53, 73 or 83

So number of distinct products =  $5 \times 6 = 30$ 

Here case III and case IV will give the same products as case II and case I respectively. ∴ Total number of distinct products = 55.

- 66. 3 Since the beads are similar and so are the two diamond pendants, it is only the number of beads between the two diamond pendants that matter. They could be 0, 1, 2, 3, 4, 5. As soon as the number of beads between the pendants become 6, the case would turn similar to the case when the number of beads was 4. Hence, in total 6 different diamond necklaces can be formed.
- 67. 4 Here, there are 12 items (10 beads and 2 pendants), out of which 2 pendants are identical. Thus, number of ways of forming a necklace

$$=\frac{1}{2}\times\frac{11!}{2}=\frac{11!}{4}$$

68. 3 7.5 + 15.5 + 10.5 + 13.0 + 13.5 + 10.5..... x - y + z = uThe LHS is the summation of two arithmetic progression. The first series has a = 7.5 and d = 3.0 and the second series has a = 15.5 and d = -2.5. The last three terms of LHS are x, -y and z.

As x, y and z, all are positive, -y; cannot be a part of the (7.5 + 10.5 + 13.5....) series. We can break the LHS as following:

 $(7.5 + 10.5 + 13.5 + \dots x + z) + (15.5 + 13.0 + 10.5 + \dots -y) = u$ 

Let there be n terms in the first bracket. The second bracket will have (n - 1) terms. We can write:-

$$\frac{n}{2}[2 \times 7.5 + (n-1) \times 3] + \frac{n-1}{2}[2 \times 15.5 + (n-2) \times (-2.50)] = u$$
  
or, n<sup>2</sup> + 101n - (u + 72) = 0  
$$\Rightarrow n = \frac{(\sqrt{10489 + 4u}) - 101}{2}$$
  
when, u = 20634,  
$$n = \frac{(\sqrt{10489 + 4 \times 20634}) - 101}{2}$$
$$\Rightarrow n = 102 \text{ and } (n - 1) = 101$$
$$\Rightarrow z = 7.5 + (102 - 1) \times (3.0) = 310.50$$
$$\Rightarrow x = z - 3.0 = 307.5$$
and y = 15.5 + (101 - 1) \times (-2.50) = -234.5
$$\Rightarrow (2x + 3y + 5z) = 11464.$$

71.4

72.4

Hence, (1) is the correct answer.



Join AP and draw  $PQ \perp AB$ .

$$\frac{2\pi}{3} = \frac{\pi \times 4^2}{2 \times 4 \times DE}$$
  

$$\Rightarrow DE = 3 \text{ units} = CF$$
  

$$\therefore CE = \sqrt{CD^2 + DE^2} = 5 \text{ units}$$
  
Let CQ = x units  

$$\therefore QB = CB - CQ = (4 - x) \text{ units}$$
  

$$\Delta CPQ \sim \Delta CEF$$
  

$$\therefore \frac{CP}{CE} = \frac{CQ}{CF}$$
  

$$\therefore \frac{4}{5} = \frac{x}{3}$$
  

$$\therefore x = \frac{12}{5} \text{ units.}$$
  

$$\therefore QB = \frac{8}{5} \text{ units}$$
  
Also,  

$$\Rightarrow \frac{PQ}{EF} = \frac{CP}{CE} \Rightarrow PQ = \frac{CP}{CE} \times EF = \frac{4}{5} \times 4 = \frac{16}{5} \text{ units.}$$
  

$$PB = \sqrt{PQ^2 + QB^2} = \sqrt{\frac{256}{25} + \frac{64}{25}}$$
  

$$= \frac{8\sqrt{5}}{5} \text{ units}$$

70. 1 Let x = number of apples, y = number of oranges and z = number of mangoes. Here, (x + y + z) = 20For favourable cases, we should have x < y < z. The total number of non-negative integral sets of (x,y,z) satisfying (x + y + z) = 20 are  ${}^{22}C_2 = 231$ . In these sets, there would be cases when two of the three variables are equal. The equal variables could be (0,0) or (1,1) or (2,2)... or (10,10). The other third variable in each of these cases would be different. Hence, each of these 11 cases would generate 3 unfavourable sets of (x,y,z). For example, (x,y,z) = (0,0,20), (0,20,0), (20,0,0) are three unfavourable cases, because we want x < y < z.

Hence, excluding these 33 cases, there are (231 - 33)= 198 sets. In these sets, there would be equal number of sets for each of (x < y < z), (y < z < x), (z < x < y), (x < z < y), (y < x < z), (z < y < x). Hence, our favourable number of cases would be  $\frac{198}{6}$  = 33 and hence the required probability =  $\frac{33}{231} = \frac{1}{7}$  $S = \left[\frac{3 \times 3 - 1 \times 2}{2 \times 3}\right] + \left[\frac{1 \times 6 - 1 \times 1}{1 \times 6}\right] + \left[\frac{2 \times 12 - 1 \times 3}{3 \times 12}\right]$  $+\left[\frac{4\times24-1\times9}{9\times24}\right]+\dots\infty$  $\Rightarrow S = \left(\frac{3}{2} - \frac{1}{3}\right) + \left(1 - \frac{1}{6}\right) + \left(\frac{2}{3} - \frac{1}{12}\right) + \left(\frac{4}{9} - \frac{1}{24}\right) + \dots \infty$  $S = \left[\frac{3}{2} + 1 + \frac{2}{3} + \frac{4}{9} + \dots \right] - \left[\frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \frac{1}{24} + \dots \right]$  $\Rightarrow S = \left(\frac{\frac{3}{2}}{1-\frac{2}{3}}\right) - \left(\frac{\frac{1}{3}}{1-\frac{1}{3}}\right)$  $\Rightarrow S = \left(\frac{3}{2} \times 3\right) - \left(\frac{1}{3} \times 2\right) = \frac{9}{2} - \frac{2}{3} = \frac{23}{6}$  $\therefore S = \frac{23}{6}$ Join PX and QY.  $\frac{PM}{PN} = \sin 30^{\circ} \Rightarrow PM = \frac{PN}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$  units  $\frac{MN}{PN} = \cos 30^{\circ} \Rightarrow MN = \frac{\sqrt{3}PN}{2} = 3 \text{ units}$  $\Rightarrow$  MX = PX - PM =  $4\sqrt{3} - \sqrt{3} = 3\sqrt{3}$  units  $\Rightarrow$  MY = QY - QM = 18 - 3 = 15 units :  $XY^2 = MX^2 + MY^2 = 15^2 + (3\sqrt{3})^2$ = 225 + 27 = 252  $\therefore$  XY =  $\sqrt{252}$  units =  $6\sqrt{7}$  units



Since, the ant is not allowed to enter inside any of the three given hexagons the path followed by the ant would be XPMNY to cover minimum possible distance. As calculated in the previous question XP = 6 units  $\therefore XPMNY = XP + PM + MN + NY$ 

= 2(PX + MN)  
= 2(6 + 
$$2\sqrt{3}$$
)  
= 12 +  $4\sqrt{3}$  units

- 74. 2 When the sum of digits is subtracted from the original three-digit number, it can only result in a two-digit number if the numbers are from 100 to 109. In each case the resulting two-digit number will be 99. If we continue the process further, we will get 81, 72, 63, 54 and so on. Note that in each case we are getting a multiple of 9. Since, one of them is a factor of the original three-digit number, the original number must be a multiple of 9. So the only possibility is 108. Among the given options only option (2) is a factor of 108.
- 75. 1 2x + 3y 2z = 0 ...(i) 4x + 3y - 3z = 0 ...(ii) 14x - 6y - 5z = 0 ...(iii) Let's take z = k (k is a real number). Putting it in equations (i) and (ii). 2x + 3y = 2k 4x + 3y = 3k $\Rightarrow x = \frac{k}{2}$  and  $y = \frac{k}{3}$

So, 
$$x = \frac{k}{2}$$
,  $y = \frac{k}{3}$  and  $z = k$ 

Now putting the value of x, y and z in terms of K, we get

 $-43 \le 2z + y + x \le 15$ 

$$\Rightarrow -43 \le 2k + \frac{k}{3} + \frac{k}{2} \le 15$$
$$\Rightarrow -43 \le \frac{17k}{6} \le 15$$
$$\Rightarrow -\frac{258}{6} \le k \le \frac{90}{7}$$

$$\Rightarrow -\frac{1}{17} \ge \kappa \ge \frac{1}{17}$$

Therefore, 21 integral values of k (from -15 to 5) are possible.

Hence, 21 integral values of z are possible.

76. 3



Here side of the equilateral triangle is 60m. In the equilateral triangle, PQ||BC because P and Q are the mid points of sides AB and AC. So, BC = 2PQ  $\Rightarrow$  PQ = 30 m

Now AE = 
$$\frac{\sqrt{3}}{2} \times 60 = 30\sqrt{3}$$
m.

'O' is the centroid, So AO : OE = 2 : 1.

 $OE = 10\sqrt{3} \text{ m}$  and  $AO = 20\sqrt{3} \text{ m}$ 

$$AF = \frac{1}{2}AE = 15\sqrt{3}m \text{ and } FO = AO - AF = 5\sqrt{3}m$$

Therefore area of  $\triangle PQO = \frac{1}{2} \times PQ \times FO$ 

$$=\frac{1}{2}\times 30\times 5\sqrt{3}=75\sqrt{3} \text{ m}^2$$

- 77. 3 The number of factors = 36 i.e.  $3 \times 3 \times 2 \times 2$ . So both the numbers are in the form  $a^2 \times b^2 \times c \times d$  or  $a^2 \times b^2 \times c^3$ , where a,b,c,d are prime numbers. HCF =  $36 = 2^2 \times 3^2$ . For the minimum possible LCM of the two numbers, the numbers should be minimum. So the numbers will be  $2^3 \times 3^2 \times 5^2$  and  $2^2 \times 3^3 \times 7^2$  and their LCM will be  $2^3 \times 3^3 \times 5^2 \times 7^2$ .
- 78.2 Discriminant of the given quadratic equation,

$$\Delta = \frac{25p^4}{(p+q+r)^2} - 24p^2k^2$$
$$= 24p^2 \left[\frac{25p^2}{24(p+q+r)^2} - k^2\right]$$

For real, distinct roots,  $\Delta > 0 \Rightarrow k^2 < \frac{25p^2}{24(p+q+r)^2}$ 

...(i)  
As, p,q and r are the sides of a triangle,  
$$\Rightarrow q + r > p$$

$$\Rightarrow \frac{p^2}{(p+q+r)^2} < \frac{1}{4}$$
...(ii)  
using, (i) and (ii)  

$$k^2 < \frac{25p^2}{24(p+q+r)^2}$$
or  $k^2 < \frac{25}{24} \times \frac{1}{4}$   
or  $k^2 < \frac{25}{96}$ 

$$\Rightarrow \frac{-5}{4\sqrt{6}} < k < \frac{5}{4\sqrt{6}}$$

or, -0.51 < k < 0.51 of the given options, only -0.45 lies inside this range.



Let, the area of the triangle ABC be denoted by  $\Delta$ 

$$\therefore \Delta BEC = \frac{\Delta}{(1+1+2)} = \frac{\Delta}{4}$$

$$\Rightarrow \Delta EGC = \frac{1}{(1+2)} \Delta BEC = \frac{1}{3} \times \frac{\Delta}{4} = \frac{\Delta}{12}$$

$$\Delta AGC = 4 \times \Delta EGC = \frac{4 \times \Delta}{12} = \frac{\Delta}{3}.$$

$$\Delta BGC = \frac{2}{(2+1)} \Delta BEC = \frac{2}{3} \times \frac{\Delta}{4} = \frac{\Delta}{6}.$$

$$\therefore \Delta ABG = \Delta - (\Delta AGC + \Delta BGC) = \Delta - \left(\frac{\Delta}{3} + \frac{\Delta}{6}\right) = \frac{\Delta}{2}$$

$$\Delta ABF = \frac{1}{(1+2)} \Delta ABD = \frac{1}{3} \times \frac{1}{2} \Delta = \frac{\Delta}{6}$$

$$(\Delta AFG + \Delta BFG) = \Delta ABG - \Delta ABF$$

$$= \frac{\Delta}{2} - \frac{\Delta}{6} = \frac{\Delta}{3}$$

$$\therefore \frac{(\Delta AFG + \Delta BFG)}{\Delta} = \frac{1}{3}$$

- 80. 3 The sum of numbers,  $(1P1.431)_8 + (231.213)_8$ = 3 x 8<sup>2</sup> + (P + 3) x 8<sup>1</sup> + 2 x 8<sup>0</sup> + 6 x 8<sup>-1</sup> + 4 x 8<sup>-2</sup> + 4 x 8 - 3 = 192 + 8 P + 24 + 2 + 0.75 + 0.0625 + 0.0078125 Or 218 + 8 P + 0.8203125 = (234.820Q125)\_{10}  $\Rightarrow$  P = 2 and Q = 3 So, P + Q = 5
- 81. 2 Let the number of chocolates with A, B, C, D and E be (a 2), (a 1), a, (a + 1) and (a + 2) respectively.  $\Rightarrow 5a = 250$  or a =50.

The following table gives the number of chocolates with A, B, C, D and E at the end of first five rounds.

	А	В	С	D	Е
Initially	48	49	50	51	52
Round 1	47	48	49	50	56
Round 2	51	47	48	49	55
Round 3	50	46	47	48	59
Round 4	54	45	46	47	58
Round 5	53	44	45	46	62

At the end of every two rounds starting from the beginning, number of chocolates with E increases by 3.

Number of chocolates with E at the end of 37 rounds =  $52 + 18 \times 3 + 4 = 110$ 

82. 4 At the end of every two rounds starting from the beginning the number of chocolates with A increases by 3. At the end of every 'odd' round, number of chocolates with A is (47 + 3n) and at the end of every 'even' round, number of chocolates with A is (48 + 3n), where n is a whole number.

Checking the options, the number of chocolates with A at the end of 36 rounds =  $48 + 18 \times 3 = 102$ . So at the end of 37 rounds the number of chocolates with A would be 101. Hence number of chocolates with A at the end of any particular round cannot be 103.

Also, among the given options only 103 cannot be expressed in the form (47 + 3n) or (48 + 3n).

83. 1 For this question, one needs to do the following steps.
(i) Numbers less than 300 relatively prime to 2 or 5 is to be found out i.e.

$$\Rightarrow 300 - \left[\frac{300}{2}\right] - \left[\frac{300}{5}\right] + \left[\frac{300}{10}\right] = 120$$

(ii) Numbers less than 300, relatively prime to 2 or 3 is to be found out, i.e.

$$\Rightarrow 300 - \left[\frac{300}{2}\right] - \left[\frac{300}{3}\right] + \left[\frac{300}{6}\right] = 100$$

(iii) Number less than 300, relatively prime to 2, 3 and 5 is to be found out

$$300 - \left[\frac{300}{2}\right] - \left[\frac{300}{3}\right] - \left[\frac{300}{5}\right] + \left[\frac{300}{6}\right] + \left[\frac{300}{10}\right] + \left[\frac{300}{15}\right] - \left[\frac{300}{30}\right]$$

 $\Rightarrow 300 - [150 + 100 + 60] + 50 + 30 + 20 - 10$  $\Rightarrow 400 - 320 = 80$ 

Therefore for all the possible integers less than 300 relatively prime to either 10 or 18 are = (i) + (ii) - (iii)  $\Rightarrow$  120 + 100 - 80 = 140

84.5 Let the three roots be  $\alpha$ ,  $\beta$  and  $\gamma$ . The second, fifth and eight terms of any geometric progression will always be the three consecutive terms of some other geometric progression as well. Hence, we can write:

$$\alpha = \frac{a}{r}$$
  
 $\beta = a \text{ and}$   
 $\gamma = ar.$ 

ĺ

also, 
$$\alpha + \beta + \gamma = \frac{-\frac{1}{\sqrt{p} + \sqrt{q}}}{1} = -\frac{1}{\sqrt{p} + \sqrt{q}}$$

or 
$$\frac{a}{r} + a + ar = \frac{-1}{\sqrt{p} + \sqrt{q}}$$
  
or  $a\left(\frac{1}{r} + 1 + r\right) = -\frac{1}{\sqrt{p} + \sqrt{q}}$  ...(i)

and 
$$\alpha.\beta + \beta.\gamma + \gamma.\alpha = \frac{1}{\sqrt{p} - \sqrt{q}}$$

$$a^{2}\left(\frac{1}{r}+1+r\right)=\frac{1}{\sqrt{p}-\sqrt{q}}\qquad ...(ii)$$

and  $\alpha.\beta.\gamma = 1$ 

or  $a^3 = 1$  or a = 1

...(iii) [:: a is real]

$$a = \frac{\sqrt{p} + \sqrt{q}}{\sqrt{q} - \sqrt{p}} \implies \sqrt{p} + \sqrt{q} = \sqrt{q} - \sqrt{p} \text{ or } p = 0$$

Hence (5) is the correct option.

85. 4 Only perfect squares have odd number of factors
 ∴ P = 49 × 64 × 81 × 100 × 121 × 144 × 169 × 196
 × 225 × 256 × 289

 $\Rightarrow$  P = 2<sup>6+2+4+2+8</sup> × 3<sup>6</sup> × N, where N is neither a multiple of 2 nor a multiple of 3.

Here highest power of 3 is 8, and highest power of 2 is more than 12. So the highest power of 12 is 8.

86. 2 
$$\frac{1}{r} + \frac{1}{s} - \frac{3}{t} = \frac{2}{5r}$$
  
(3s-t) 3

-st r = -5

Now, st must be among 5, 10, 15, 20, 25, 30, 35, 40, 45. Only, s = 2, t = 5, r = 6and s = 5, t = 6, r = 2 are found to be valid. Therefore, (r + s - t) = 3 or 1.

Therefore there are two possible values of (r + s - t)



Let us define 'x' and 'y' as the number of members belonging to 'all 4' and 'exactly 3, categories respectively. Similarly (a + b) denotes the number of members belonging to 'exactly 2' categories.

We can define a term called 'excess data' as the difference between the sum of number of members belonging to 4 individual categories and the actual number of members in the club.

This 'excess data' will be absorbed with the increasing number of members belonging to exactly 2, 3 or 4 categories.

# Case I:

Here excess data is (220 - 170) = 50. On the basis of excess data, we can maximize x. Assume 'x' to be 1, then 3 will be subtracted from the excess data. So the maximum value of x can be 16. Now assume 'y' to be 1, then 2 will be subtracted from the excess data. Therefore excess data will be zero when x = 16 and y = 1.

Hence members belonging to at least three categories = 16 + 1 = 17.

### Case II:

Maximum value of x = 16. But if we put y = 0, a = b = 1, even then the excess data gets absorbed.

In this case there will be no member belonging to 'exactly 3' categories and 2 members belonging to 'exactly 2' categories. In this case, members belonging to at least 3 categories = 16 + 0 = 16Hence, (5) is the correct choice.

88. 1 Here x is 10, then excess data is (50 - 30) = 20. To maximize the number of members belonging to Physically challenged category, 10 members can belong to three categories except Physically challenged. Therefore excess data will be absorbed when x = 10 and y = 10. Hence maximum number of members belonging to only Physically challenged category = (50 - 10) = 40.

89. 5 Since 'a' when divided by 'N' gives remainder 4, then a = Nx + 4. ('x' is a natural number) Similarly, b = Ny + 3. ('y' is a natural number) Given that 'b' is twice of 'a'. Therefore, b = 2Nx + 8. Since b when divided by N gives remainder 3, N has to be 5.

> Therefore, a = 5x + 4 and b = 5y + 3. ∴ 100a + 11b = 100(5x + 4) + 11(5y + 3) = 500x + 55y + 433 = 55(9x + y) + 5x + 433. So, the remainder will be (5x + 433) when (100a + 11b) is divided by 55. 5x + 433 = 5(x + 86) + 3.

Dr, 
$$\frac{5(x+86)+3}{55} = \frac{x+86}{11} + \frac{3}{55}$$
.

Since the remainder when (100a + 11b) is divided by 55 is 23, therefore one of the possible of values x = 6. [Because when x = 6, the remainder will be  $4 \times 5 + 3 = 20 + 3 = 23$ ]

Similarly, we can conclude that x = 11m - 5, when 'm' is a natural number

 $100 \le 55m - 21 \le 999$ 

Number of possible values of m is 16 (from m = 3 to m = 18).

.: 16 values of 'a' are possible

90. 4 
$$S = 6a^{2} - 4a + \frac{12a}{b} + \frac{6}{b^{2}} - \frac{4}{b}$$
$$\Rightarrow S = 6\left(a^{2} + 2\frac{a}{b} + \frac{1}{b^{2}}\right) - 4\left(a + \frac{1}{b}\right)$$
$$\Rightarrow S = 6\left(a + \frac{1}{b}\right)^{2} - 4\left(a + \frac{1}{b}\right)$$

From the given expression  $b(a-1) = \sqrt{3}b - 1$ 

We can get that 
$$a + \frac{1}{b} = \sqrt{3} + 1$$

Now putting the value of 
$$\left(a + \frac{1}{b}\right)$$
, we get  
 $S = 6\left(\frac{1}{2} + 1\right)^2 = 4\left(\frac{1}{2} + 1\right)$ 

$$\Rightarrow S = 6(4 + 2\sqrt{3}) - 4(\sqrt{3} - 4) = 20 + 8\sqrt{3}$$